

Lepton-Pair Production in Virtual Compton Scattering off the Proton

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Abstract

We show that lepton-pair production in Virtual Compton Scattering offers, through interference with the well-known Bethe-Heitler process, a sensitive probe to learn the longitudinal response of resonances and the electromagnetic nucleon form factors. This interference can be measured directly in terms of an asymmetry. The role of off-shell effects in the N-N- γ vertices is investigated as well. An additional N-N- γ - γ contact term in the amplitude, included to ensure gauge invariance of the model, cancels a substantial part of the off-shell effects.

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We investigate Virtual Compton Scattering (VCS) in the time-like region of the photon momentum k^2 , the process where the incoming photon is real with a virtual outgoing photon, decaying into an electron-positron pair. One important reason for investigating VCS is to extract the nucleon form factor in the ‘unphysical’ region, $k^2 < 4M^2$ (M is the nucleon mass), where no data exist. In this letter physics is addressed related to the more general electromagnetic structure of the nucleon, such as off-shell effects in the N-N- γ vertex and the longitudinal response of nucleon resonances. The main complication in VCS is the huge ‘background’ due to the well known Bethe-Heitler (BH) process. In Ref. [1] the electromagnetic vertex in the unphysical region has also been considered where it was shown that, due to this large BH contribution, only at large opening angles of the lepton pair information on the time-like form factor could be extracted. As shown in this letter the interference with the BH-background can be exploited to make VCS a sensitive measure for nucleon structure.

Figure 1: The tree-level Feynman graphs included in the calculation where diagrams a and b correspond to the Bethe-Heitler process. Circled dots represent vertices where off-shell effects are considered, double lines represent nucleon resonances, and the dashed line is a π^0 .

Following an idea originally proposed in [2] and more recently investigated in [3] the BH-process can be used to ones advantage. The lepton pair emerging from the BH process interacts with two photons (see Fig. 1a,b) and thus has positive charge-conjugation parity (C-parity) [4]. The lepton pair from the nuclear process (Fig. 1c-h) interacts with a single (virtual) photon and thus has negative C-parity. The different symmetry of the two matrix elements under C-parity implies that of the three terms in the differential VCS cross section,

$$d\sigma(e^+, e^-) \propto |\mathcal{M}_{BH} + \mathcal{M}_N|^2 = |\mathcal{M}_{BH}|^2 + |\mathcal{M}_N|^2 + 2\text{Re}(\mathcal{M}_{BH}\mathcal{M}_N^*) , \quad (1)$$

the first two terms on the right-hand side are symmetric under the interchange of e^+ and e^- while the last one is anti-symmetric. The Bethe-Heitler-nuclear (BH-N) interference can thus be measured directly through the asymmetry

$$A_{BH-N} = \frac{d\sigma(e^+, e^-) - d\sigma(e^-, e^+)}{d\sigma(e^+, e^-) + d\sigma(e^-, e^+)} = \frac{2\text{Re}(\mathcal{M}_{BH}\mathcal{M}_N^*)}{|\mathcal{M}_{BH}|^2 + |\mathcal{M}_N|^2} . \quad (2)$$

where $d\sigma(e^-, e^+)$ is the cross section under the same kinematic conditions as $d\sigma(e^+, e^-)$ with only an interchange of the leptonic charges. In the following the discussion is limited to this asymmetry.

To have a clear link to the nuclear process we have chosen as kinematical variables not the momenta of the electron (k_1) and positron (k_2), but instead k^2 (where $k = k_1 + k_2$, the momentum of the virtual photon in the diagrams in Fig. 1c-h), θ_k (the angle between \vec{k} and the incoming real photon), θ_d , (the polar angle between \vec{k} and $\vec{k}_d = \vec{k}_1 - \vec{k}_2$), and ϕ_d (the azimuthal angle between the reaction plane and that of the e^+e^- -pair). In these variables interchanging the leptonic charges corresponds to changing \vec{k}_d to $-\vec{k}_d$. It should be noted that due to reflection symmetry with respect to the reaction plane the asymmetry vanishes for $\phi_d = 90^\circ$, the results are henceforth quoted for the in-plane conditions $\phi_d = 0^\circ$. For a polarized real photon or a polarized target this reflection symmetry is broken [3].

In the calculations only the Δ - and the Roper- (P_{11}) resonances have been included. The width of the resonances is generated in the unitarized K-matrix approach through the coupling to the one-pion decay channel. Parameters in the $N-\gamma$ -resonance vertices have been chosen to obtain a best fit to the real Compton-scattering cross sections at photon energies $E_\gamma = 150 \div 370$ MeV. For details of the model and the default parameters we refer to Ref. [5] (parameter set#2).

In the present discussion of VCS we will focus on terms in the electromagnetic vertices that cannot be studied in real Compton scattering. Since their effects generally increase with increasing k^2 , we limit the present discussion to k^2 close to the kinematical limit ($\sqrt{s} = \sqrt{k^2} + M$). Even in these kinematical conditions the nuclear matrix element is still dominated (to more than 90%) by transverse-polarized virtual photons, the longitudinal response plays a much smaller role than expected. Inclusion of nucleon resonances in the calculation is thus important. Since only the lowest two nucleon resonances are included in the model we performed calculations at $E_\gamma = 500$ MeV in the lab, corresponding to $\sqrt{s} = 1.349$ GeV, and at $\sqrt{k^2} = 406$ MeV (photon invariant mass for the nuclear contribution). At this energy also higher resonances such as especially the D_{13} may be important for the detailed results, but not for the general discussion and conclusions in this letter as we have verified. For the k^2 dependence of the nucleon and $N-\gamma$ -resonance transition form factors we used the dipole fit for negative k^2 and Vector Meson Dominance (VMD) model for positive k^2 . Note that even though the longitudinal response contributes little to the nuclear matrix element, it strongly affects the asymmetry.

The VCS matrix element depends strongly on the parameters in the $N-\gamma$ -resonance vertices. For simplicity and as an example we limit ourselves to the discussion of the longitudinal part of the $N-\Delta-\gamma$ vertex [6], proportional to G_3 , which does not contribute to real Compton scattering. In Fig. 2 the curves labelled ‘ G_3 ’ show the calculated asymmetry A_{BH-N} for $G_3 = 20$ which can be compared with the calculations using default parameters ($G_3 = 0$, curves labelled ‘norm’). At backward angles (lower panel of Fig. 2) the effect of G_3 is not just an overall enhancement of the interference pattern but introduces a change in the structure. This is due to a changing ratio of longitudinal v.s. transverse response which is reflected in a different dependence [7] on the opening angle θ_d . We also find that the asymmetry depends strongly on the off-shell parameter z_3 [8]. The results shown in Fig. 2 have been obtained for $z_3 = -0.5$, at $z_3 = 0$ the dependence on G_3 almost vanishes.

It is known that the $N-N-\gamma$ vertex for off mass-shell nucleons has a more complicated structure [9, 10, 11, 12] than that for the free nucleon. The corresponding form factors

Figure 2: Asymmetry in the center-off-mass system calculated as discussed in the text. In the top panel the opening angle of the lepton pair, θ_d , is kept fixed, while the direction θ_k of the lepton pair momentum varies. In the lower two panels the asymmetry is plotted as function of θ_d at fixed θ_k . The curves ‘ G_3 ’ show the contribution of the longitudinal coupling to the Δ -resonance, the curves ‘ ∂F_1 ’ and ‘ F_1^- ’ display the effect of the inclusion of the off-shell form factors in the N-N- γ vertex. The effect of switching off the contact term Eq. (8) (for the calculation ‘ F_1^- ’) is shown by the dashed-dotted curves. The effect of the q^2 -dependence in the form factors can be seen from the curves ‘no VMD’.

depend on all Lorentz invariants that can be formed from the momenta at the vertices, not only the photon invariant mass q^2 . In particular we will consider the dependence on the invariant mass W of the intermediate nucleon in the diagrams Fig. 1c,d. For sake of simplicity we limit ourselves to a half-off-shell vertex of the structure

$$\begin{aligned}\Gamma_{NN\gamma}^\mu &= \hat{e}\gamma^\mu + \tilde{F}_1(q^2)\mathcal{O}_1(p')(q^\mu q^\nu - q^2\gamma^\mu)\mathcal{O}_1(p) \\ &+ F_2(q^2)\mathcal{O}_2(p')i\frac{\hat{\kappa}}{2M}\sigma^{\mu\nu}q_\nu\mathcal{O}_2(p)\end{aligned}\quad (3)$$

based on a Taylor-series expansion, with

$$\mathcal{O}_i(p) = 1 + (A_i + B_i)\frac{p}{M}\left(\frac{p}{M} - 1\right), \quad (4)$$

and $q = p' - p$ ($\hat{e} = 1(0)$ for the proton (neutron)). Here $F_1(q^2) = \hat{e} - q^2\tilde{F}_1(q^2)$ and $F_2(q^2)$ correspond to the usual Dirac and Pauli form factors in the VMD model. The off-shell dependence is introduced by the operators \mathcal{O}_i normalized such that

$$\mathcal{O}_i(p)u(p) = u(p), \quad (5)$$

when $u(p)$ is a positive-energy solution of the free Dirac equation. The off-shell effects are illustrated through the relations

$$\frac{\partial \mathcal{O}_i(p)}{\partial p}u(p) = \frac{A_i + B_i}{M}u(p) \quad (6)$$

$$\mathcal{O}_i(p)v(p) = (1 - 2(A_i - B_i))v(p), \quad (7)$$

where $v(p)$ is a negative-energy state. Choosing $A_i = B_i = 0$ corresponds the conventional form for the free nucleon vertex.

The diagrams in Fig. 1c-g satisfy gauge invariance if off-shell effects are absent. Inclusion of these leads to a violation of current conservation and thus requires additional terms in the reaction amplitude even though the vertices in Eq. (3) obey the Ward-Takahashi identity for the reducible vertex. Using constraints imposed by the current conservation with respect to initial and final photons (see [13]) one can construct an effective N-N- $\gamma\gamma$ vertex (Fig. 1h) to correct gauge invariance. The corresponding vertex (called here the contact term) for the off-shell dependence in Eq. (4) takes the form

$$\begin{aligned}\Gamma_{NN\gamma\gamma^*}^{\mu\nu} &= 2\hat{e}\tilde{F}_1(k^2)\frac{(A_1 - B_1)}{M}\left(k^2g^{\mu\nu} - k^\mu k^\nu\right) \\ &+ 2\hat{e}\tilde{F}_1(k^2)\frac{B_1}{M^2}(p + p')^\mu(k^2\gamma^\nu - k^\nu k^\mu) \\ &+ \hat{e}\frac{\hat{\kappa}}{2M}\frac{(A_2 - B_2)}{M}\left[(\gamma^\mu q^\nu - \gamma^\nu q^\mu) + F_2(k^2)(\gamma^\mu k^\nu - \gamma^\nu k^\mu)\right] \\ &+ 2i\hat{e}\frac{\hat{\kappa}}{2M}\frac{B_2}{M^2}\left[-(p + p')^\nu\sigma^{\mu\rho}q_\rho + F_2(k^2)(p + p')^\mu\sigma^{\nu\rho}k_\rho\right].\end{aligned}\quad (8)$$

Here the index μ (ν) and momentum q (k) refer to the real (virtual) photon. One should keep in mind that the off-shell effects may be different for different representations of the same theory. This has been shown for the case of real Compton scattering off the pion [14] and in a different context in [15]. The off-shell effects addressed in the present paper

are defined within a particular representation for the effective Lagrangian. Note that the structure of this contact term is not unique and ambiguities in its structure may be as important as the effect of the off-shell dependence itself.

The physics of the contact term can be understood as follows. Introducing an explicit nucleon-momentum dependence in the vertex corresponds in coordinate space to creation and annihilation of the nucleon at different positions, i.e. the introduction of some general finite size effects. When the nucleon is charged this corresponds to violation of current conservation. An additional contact term should therefore be included corresponding to the coupling of the photon to the current which was not accounted for. Alternatively, in a one-pion loop model for form factors, off-shell effects arise through the fact that the (virtual) photon can couple to both the nucleon and the pion in the loop. Inserting this vertex in a calculation where one deals with two photons, like Compton scattering, one should in addition take into account that the other photon couples also to the charged particles in the loop. This effect leads to an effective $N\text{-}N\text{-}\gamma\text{-}\gamma$ vertex.

The VCS matrix element depends strongly on off-shell dependences in both the Dirac and Pauli terms in Eq. (3). Since the effect of the Pauli term can also be investigated in real Compton scattering we will limit the present discussion to the Dirac term and henceforth take $A_2 = B_2 = 0$. One can distinguish two cases:

∂F_1 : $A_1 = B_1$ in Eq. (3) corresponds to the equal coupling for negative- and positive-energy states (see Eq. (7)) and a non-vanishing derivative of the coupling at the on-shell point (see Eq. (6)). The calculation for $A_1 = B_1 = 1$ is labelled ‘ ∂F_1 ’ in Fig. 2.

F_1^- : $A_1 = -B_1$ in Eq. (3), in contrast, corresponds to a the vanishing derivative and a different coupling for positive- and negative-energy states (compare Eq. (5) and Eq. (7)). The calculation for $A_1 = -1, B_1 = 1$ is labelled ‘ F_1^- ’ in Fig. 2.

From Fig. 2 it is apparent that both dependences give rise to comparable effects in the asymmetry of about 5% in A_{BH-N} . Under conditions further away from the kinematical limit, which means smaller values of the photon invariant mass at fixed \sqrt{s} , the effects of these terms becomes even less.

As remarked, the construction of the contact term Eq. (8) is not unique and we therefore compare results with a calculation in which the contact term is switched off. This leads to a violation of current conservation, however, it may give an estimate of the importance of this contact term. In the ‘ ∂F_1 ’ calculation the contact term does not affect the results, while in the ‘ F_1^- ’ calculation it appears responsible for suppressing most of the off-shell dependence (compare curves ‘ F_1^- ’ and ‘ F_1^-, NC ’ in Fig. 2).

The calculations of ref. [12], based on a one-loop model and VMD, indicate that the values of the parameters A_1 and B_1 are of the order of 0.1, much smaller than the values of the off-shell parameters A_2 and B_2 that enter in the Pauli form factor which are of the order of 1/4. As a result for realistic cases one expects the off-shell effects to be negligible in the asymmetry.

The k^2 dependence of the form factors is also reflected in the asymmetry. Neglecting this dependence in all vertices changes the results considerably (see Fig. 2, curves labelled ‘no VMD’). At the backward angle the interference pattern is affected which offers the possibility to disentangle the k^2 dependence from off-shell effects. Therefore measurement

of the asymmetry allows for a verification of the VMD model in the region $k^2 < M^2$, which could be complementary to the suggestion in [1] to study form factors in the dilepton mass spectra under symmetrical conditions ($\theta_k = 0^\circ$ or 180° , $\theta_d = 90^\circ$). However, to obtain information on the form factors in the interesting region $k^2 \approx m_\rho^2$ (m_ρ is the mass of the ρ -meson) one has to study VCS at higher photon energies where, of course, contributions of the higher nucleon resonances should be accurately taken into account.

It can be concluded that the asymmetry in VCS offers an interesting tool to study the longitudinal response of nucleon resonances and electromagnetic form factors of the nucleon. In view of the results of [2] we believe that in modern experiments it should be possible to distinguish the different effects. The most interesting region is at backward virtual-photon angles close to the kinematical limit where from the θ_d dependence it appears to be possible to verify the VMD model and to separate the longitudinal response of resonances.

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